Synopsis

“The study of Banach Contraction Mapping Principle, fixed point theorems and its applications”

• Submitted to •

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• **For the degree of** •

Doctor of philosophy (Ph.D.)

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***Abstract of Proposed Work Plan*:**

: As stated by Brown, R. F., ed “In mathematics, a fixed-point theorem is a result saying that a function F will have at least one fixed point (a point x for which F(x) = x), under some conditions on F that can be stated in general terms [1].” Moreover, a contraction mapping has the one fixed purpose. Under this, the Banach fastened-point theorem states that each contraction mapping on a nonempty complete topological space which contains a distinctive fixed purpose, that may be for any x in M the iterated perform sequence x, f (x), f (f (x)), f (f (f (x))). It converges to the secured purpose. This idea is extremely helpful to iterate the performance systems in which the contraction mappings are typically used. Banach's fixed-point theorem applies as a proof of the existence of solutions in normal differential equations as well as it is employed as one proof of the function theorem. Therefore, the current work is an attempt of the Generalization of Banach Contraction Mapping Principle and fixed point theorems.

The researcher will try to find out the study of of Banach contraction Mapping Principle, fixed point theorems and its application.

I. Introduction:

The Study of Metric space is one of the best and powerful tools in the use of fixed point Theorems. The most of researchers in the subject of nonlinear Analysis and Topology are at present studying at the wide range. Now-a-days, there is anavailability of wide range of literature in this direction.

In 1906, the Mathematician M. Frechet firstly introduced the notion of metric space. Further development of this notion is developed by the Hausdorff. In 1914, further studies are continued by various Authors and develop the metric space and some fixed point theorems under different contraction conditions which are based on the more useful formulation for fixed point theorems.

The various topological spaces such as Metric space, complete metric space, Banach space, Hilbert space, locally convex and Hausdorff space etc, have been employed in the formulation of the fixed point theorems for various classes of contraction mappings.

Here researcher finds the need to study the non-expansive mappings, strict fixed point theorems. Ciric's fixed point theorems, khamsi's fixed point theorems, Brouwer fixed point theorem, fixed point theorems for multi valued contractive mappings, contraction mapping principle, Edelstain fixed point theorem, kranosekii fixed point theorem, Quasi non expansive mappings, Hahn Banach theorem. Bnyd and wong's fixed point theorem, multivalued mappings for fixed point in Complete metric space'. Researcher also studies some common fixed points.

Researcher wants to study the above theorems and their useful application. Also the researcher will modify the fixed point theorem in Metric space and Banach space, also the researcher will study some fixed point theorems for proving fixed point, and also we are going to study

Banach contraction mapping principle and fixed point theorems with their applications.

In current research study, Researcher will also going through the study of fixed point with a well-known classical result about the existence and uniqueness of fixed point of contrition that is a function F from Metric space X to itself satisfying the condition

$$d(F(x),F(y)) \leq r\, d(x,y)$$

For all x,y in X.where r is such that, $0 \leq r \leq 1$

We shall give an application of this result to the solution fixed point theorem by using the some contractive conditions.

Review of Literature and Development in the subject (Previous work done in the relevant area):

Recently, Rhoades, Andrian in, Dolhare and Dhage, introduced a notion of a new structure of fixed point theorems for the mapping satisfying certain contraction type conditions. Further Metric space developed by Dolhare Section that is celebrated in Banach , Dolhare and Dhage and Asad N.A. for contracting principle in boundary metric space follows from the contraction mapping principle in metric space as a special case. Similarly, it is hoped that several fixed point theorems in ordinary metric space as a special cases. Therefore, it is interested to study the fixed point theorems for various classes of mapping in metric as well as Banach space. Apart from these following are few studies based on the contraction of fixed point theorem and contraction mapping principles.

Objectives of Research:

Objective of research study is eight folds.

1) To discuss the fixed points in Metric Space.

2) To discuss the fixed points in non-expansive mapping.

3) To discuss the fixed points for contraction mapping in Metric Space.

4) To discuss the fixed points for contraction in Banach Space.

5) To study of fixed points in D-Metric space.

6) To study of fixed points in G-Metric space.

7) To study the generalization of fixed point theorems in Banach space and Metric Space.

8) To study the applications of fixed point theorems in Metric and Banach Space.

Importance of study/ Society application:

International Status:

Many mathematicians have declared the operating system, within the metric fastened purpose theory. Banach, R. Kann, L.B. Cric,Rhoades have worked in fastened purpose theory. In currently decades, Zead Musiala and Brailey slips developed terribly nice area i.e. D-Metric area & Generalized many fixed points additionally, Abbas M., Sintunvarat, Aydi, Karpinar worked in the operating system during this field,

National Status:

Indian Mathematicians additionally operating within the fixed point theory in metric space, complete metric space,D-metric space, G-metric space. In those studies, Chaudhury B.S., R.K. Dharadwaj, R. Kanan, R. K. Sharma, Bharti Choure, P, Das, Laxmikanta Devi, C.T. Aage, S. Bedre & V. Dapkar B.C.Dhage, U.P.Dolhare contributed some leads to this knowledge base.

Significance of Study:

Fixed points theory incorporates wide applications in several fields of science. In computing to various arguments, there is the most outstanding use of fixed points in Lattice theory. Fixed purpose Theory is additionally employed in political economy and scientific theory. The kakutani fastened purpose Theorem are often accustomed prove the minimal theorem within the theory of zero-sum games. John Ogden Nash used this theorem in existence of Nash equilibrium in each finite game with mixed strategies for any range of players. This work later awarded him as a Nobel Prize Winner in political economy. Fixed points theory has several applications in arithmetic itself, to find resolution of differential equations to fastened solutions of integral equations etc.

Methodology and Stages in Research Work:

First Stage: In the first stage of my research work , Researcher will try to build up the necessary background to understand the area of my research work and for this purpose; Researcher will read the useful research text books.

Second Stage: Survey of the research papers to collect the literature related to my research problem.

Third Stage: Generalization of new theorems which will be new contribution in Research.

Fourth Stage: Fourth stage will be the new results and conclusion.

Fifth Stage: Bibliography and Reference Books.

Conclusion:

Thus, The Banach contraction principle may be a very hip tool in determination existence issues in several branches of mathematical analysis. The study of fixed points for ambiguous contraction maps victimization the Hausdorff metric was initiated by Nadler in 1969; United Nations agency extended the Banach contraction principle to set-valued mappings. Since then, several authors have studied fixed points for set-valued maps.

Year wise plan of work and Target to be achieved:

First year:-

From different research canters, libraries and on Internet and we will study the basics related to the research problem. Researcher will attend and present some papers related to problem in some Journals and conferences.

Second year:- To attend the conference, workshop and seminars in relevant topic, and collect research papers and study them

Third Year: - In this stage we will study some fixed point theorems in metric spaces & its generalizations in G-metric space & try to generalize them.

Fourth Year: To publish research papers in reputed journals & Co point research work & plan to submit thesis to university.

References :

[1] Brown, R. F., ed. (1988). Fixed Point Theory and Its Applications. American Mathematical Society. *ISBN 0-8218-5080-6*

[2] Dolhare U.P., Dhage B.C., Adrian petrusel.

Some common fixed point theorems or sequence of NonselfMultivated operations in Metrically convex D-metric space.Maths,Fixed point Theory.**International Journal, Romania,Value 4.No.(2003),143-158**

[3] Dolhare U.P. Nonlinear self-mapping and some fixed point theorems in D-metric spaces. Published in Second International conference of Nonlinear systems.Modeling, Simulation and application Dec.2006.

[4] Dolhare U.P. Generalization of Banach contraction mapping principle Math.Soc. Abp. 1 (2010), 09-12.

[5] Assad N.A. Fixed valued theorems for set valued transformations on compactset. **Bull. Un. Math. (4). (1973) L-7**

[6] Dolhare U.P. Some fixed point theorems in D-metric Spaces published in**Nantional Conf. on Nonliner Analysis and application March 24**

[7] AndrianConstantin On some fixed point theorems in metric space. Univ. Novmsadu.Zb. Rad. Prirod Mat. **Fak. Ser. Mat. 24, 2 (1994), 9-21**

[8] Cric L.B. On a family of contructive maps and fixed points **Publication DE L institute MathematiqueNouvello serve tome 17 (31), 1974, PP-45-51.**

[9] Rhoades B.E. A comparison of various definitions of contractive mappingsTransactions of the **American Mathematical society volume 226,1977.**

[**10] S.N. Mishra and Rajendra Pant** Generalization of some fixed point theorems in ultra-metric spaces. Adv. **Fixed point theorem, 4 (2014),No. I, 41,-47 ISSN 1927- 6303**

[11] G. Jung& and B.E. Rhoades Fixed points for set valued functions without Continuity. **Indian J. Pure appl. Math 29 (3) :227-238. March 1998.**

[12] M.S. Khan and M. Swatch Fixed point theorms for generalized contraction **Indian J. Pure appl. math 15 (9) :984 - 990. September 1984.**

[13] R.K. Sharma, BhartiChowre, R. K.Bhardwaj Fixed point result in Banach spaces **Mathematical theory and modeling Vol.4 No.1.2014**

[**14] A Branciari** A fixed point point theorem for mappings satisfying a general contractive condition of integral type **IJMMS(2002) 531-536 PII-Hindawi Publishing Corp.**

[15]Richard S. Palais "A simple proof of the Banach contraction principle" J. Fixed point theory appl. 2 (2007), 221-223 (c) 2007. Birkhauser, Verlag Basel/ Switzerland

[16] Browder F.E.: Non-expeansive non-linear operators in Banach Space.

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